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Robust Air-Traffic Control Using Ground-Delays and Rerouting of Flights

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In this paper we demonstrate that finite horizon optimization of ground-delays and rerouting can provide robustness to the performance of the United States National Airspace System, in the event a center goes down. Our analysis is based on a linear system approximation of the National Airspace System. We use the linear model to analyze the degrading effect of a loss of center on the performance of the air-traffic control. We employ ground-delays and rerouting of flights as mitigation tools and propose optimal sequences of these control actions to restore the performance of the National Airspace System. The optimal sequence is determined by solving a finite horizon optimal control problem. Historical data in the available literature are used to derive the linear approximation of the National Airspace System. From our analysis we observe that both ground-delays and rerouting are able to successfully restore the performance of the National Airspace System. We did not observe significant difference in performance between these two methods. However, the computational complexity associated with rerouting is significantly more than that for ground delays. The simulation plots demonstrate that finite horizon optimization for determining optimal ground-delay or rerouting strategies can mitigate the effects of a center going down in the National Airspace System.

I. Introduction

THE United States National Airspace System (NAS) has become a primary topic of research, over the years. Uncertainties due to increase in local traffic, local flight delays, adverse weather conditions, maintenance and security issues affect the operation of the NAS. Several models to determine how these uncertainties affects the flow of aircrafts in NAS are currently being developed. Of major interest is how loss of a center affects the flow of traffic in the rest of the NAS. To address this problem, new and efficient methods to control traffic in US NAS are been developed. There are several modeling approaches to capture dynamics of the NAS.

In the deterministic setting, there are several approaches. There are methods that model aircraft to aircraft interactions [1] and are used for capturing the local NAS dynamics. The dynamics of the NAS at a national level is captured using aggregate level model, where the complexity due to large number of aircrafts is reduced by considering traffic densities and their flows [2]. The aggregate level uses the structure of the NAS as a whole. It has been used to model the flows of aircrafts between all the 23 centers of the US NAS, as a linear dynamical system [3]. Methods based on Eulerian approach have also been proposed [4], where the NAS is partitioned into surface elements (SELs), based

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on a latitude–longitude grid. Flows in and out of the SELs are assumed to be through eight traffic flow directions. A linear system is then constructed using the traffic inflow and outflow information through SELs.

In the probabilistic setting, flow of aircrafts are assumed to be defined by probability density functions. This is motivated by the fact that probabilistic uncertainty in aircraft demand exists at various sectors of the NAS. In this framework, aircraft arrivals and departures are represented by possion processes and flows of aircrafts as random variables [5]. This has led to the development of queueing models, which have been used to assess sensitivity of the NAS to probabilistic uncertainties [6]. Queuing models have also been used in air traffic flow management problems [7].

In this research work, we use the aggregate level model of the US NAS to determine optimal strategies for the centers that will minimize the effect of a center going down on the performance of the NAS. We consider ground-delay and rerouting as two options for each center and formulate the control problem using finite horizon optimal control theory. For the strategy based on ground delays, we ground the vehicle to the originating center, if the destination center goes down. The excess traffic is then regulated using takeoffs and landings. For the rerouting strategy, we ensure that the aircrafts are only routed through the neighboring centers of the center that goes down, based on the connectivity of NAS. Our goal is to minimize the deviation from the number of aircrafts that existed in the airspace before loss of a center.

II. Modeling of the NAS in Aggregate Level

We model the NAS as a linear system in an aggregate level [2]. The continental US airspace consists of 20 centers, within which traffic flow differs in different time of the day. We use existing model of the NAS [8], consisting of 20 states corresponding to the number of aircrafts in air at every center, and additional 20 states that correspond to the number of aircrafts that are on the ground at every center. The dynamics is modeled by a discrete time linear system, where each time step is of an hour. The model conserves the total number of aircrafts.

Let N be the total number of centers under consideration, $x_a(k) \in \mathbb{R}^N$ be the state variable representing number of aircrafts in air, $x_g(k) \in \mathbb{R}^N$ be the state variable representing number of aircrafts on the ground, $u(k) \in \mathbb{R}^N$ be the control variable representing the number of take offs from the centers, and $l(k) \in \mathbb{R}^N$ be the control variable representing the number of landings at given center. These variables are all functions of the discrete time step, represented by k. The dynamics of $x_a(k)$ and $x_g(k)$ can be written as

$$x_a(k+1) = A_a(k)x(k) + u(k) - l(k), \tag{1}$$

$$x_g(k+1) = x_g(k) + l(k) - u(k), \tag{2}$$

where $A_a \in \mathbb{R}^{N\times N}$. The ground state of a center can only have transition to the corresponding center, hence the corresponding "A" matrix for x_g is taken to be identity. The transition between air and ground is controlled through l and u.

The system in Eqs. (1) and (2) can be derived [8] for a three-center model as follows. The associated dynamical system has six states. Let ρ_{ij} represent the number of aircrafts leaving center i and going to center j during the aggregation time interval T. The outflow rate from center i to j is given by ρ_{ij}/T. Thus, the number of aircrafts that go from center i to center j over time interval Δt is $\rho_{ij}\Delta t/T$. If the average number of aircraft in center i during the time period T is denoted by $\bar{x}_i$, the fraction of aircraft that go from center i to j during the time interval is given by $\rho_{ij}\Delta t/\bar{x}_i T$. This quantity also represents the transition probability. Using these steps for aggregation, the following define the aircraft flow dynamics of aircraft in a three-center problem.

$$x_{a_1}(k+1) = x_{a_1}(k) + \frac{\rho_{21}\Delta t}{\bar{x}_2 T}x_{a_2}(k) + \frac{\rho_{31}\Delta t}{\bar{x}_3 T}x_{a_3}(k) - \frac{\rho_{12}\Delta t}{\bar{x}_1 T}x_{a_1}(k) - \frac{\rho_{13}\Delta t}{\bar{x}_1 T}x_{a_1}(k) + u_1(k) - l_1(k), \tag{3a}$$

$$x_{a_2}(k+1) = x_{a_2}(k) + \frac{\rho_{12}\Delta t}{\bar{x}_1 T}x_{a_1}(k) + \frac{\rho_{32}\Delta t}{\bar{x}_3 T}x_{a_3}(k) - \frac{\rho_{21}\Delta t}{\bar{x}_2 T}x_{a_2}(k) - \frac{\rho_{23}\Delta t}{\bar{x}_2 T}x_{a_2}(k) + u_2(k) - l_2(k), \tag{3b}$$

$$x_{a_3}(k+1) = x_{a_3}(k) + \frac{\rho_{13}\Delta t}{\bar{x}_1 T}x_{a_1}(k) + \frac{\rho_{23}\Delta t}{\bar{x}_2 T}x_{a_2}(k) - \frac{\rho_{31}\Delta t}{\bar{x}_3 T}x_{a_3}(k) - \frac{\rho_{32}\Delta t}{\bar{x}_3 T}x_{a_3}(k) + u_3(k) - l_3(k), \tag{3c}$$

where $x_a := [x_{a_1}\ x_{a_2}\ x_{a_2}]^{\mathrm{T}}$, $u := [u_1\ u_2\ u_3]^{\mathrm{T}}$, $l := [l_1\ l_2\ l_3]^{\mathrm{T}}$. Equation (3a) can be simplified to be

$$x_{a_1}(k+1) = \left(1 - \frac{u_{12}\triangle t}{\bar{x_1}T} - \frac{u_{13}\triangle t}{\bar{x_1}T}\right) x_{a_1}(k) + \left(\frac{u_{21}\triangle t}{\bar{x_2}T}\right) x_{a_2}(k) + \left(\frac{u_{31}\triangle t}{\bar{x_3}T}\right) x_{a_3}(k) + u_1(k) - l_1(k). \tag{4}$$

By simplifying the remaining equations, we can write the dynamics in a compact form as

$$\begin{pmatrix} x_{a_1}(k+1) \\ x_{a_2}(k+1) \\ x_{a_3}(k+1) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{pmatrix} x_{a_1}(k) \\ x_{a_2}(k) \\ x_{a_3}(k) \end{pmatrix} + \begin{pmatrix} u_1(k) - l_1(k) \\ u_2(k) - l_2(k) \\ u_3(k) - l_3(k) \end{pmatrix}, \tag{5}$$

where a_{ij} are determined from Eqs. (3a) to (3c). Equation (5) is in the form of Eq. (1).

For ground states $x_g := [x_{g_1}\ x_{g_2}\ x_{g_3}]^{\mathrm{T}}$, we have

$$x_{g_1}(k+1) = x_{g_1}(k) + l_1(k) - u_1(k), \tag{6a}$$

$$x_{g_2}(k+1) = x_{g_2}(k) + l_2(k) - u_2(k), \tag{6b}$$

$$x_{g_3}(k+1) = x_{g_3}(k) + l_3(k) - u_3(k), \tag{6c}$$

where $x_{g_i}(k)$ represents the number of aircrafts in the ground for the ith center, at the kth time step.

These equations can be combined together and written in a compact form as

$$\begin{pmatrix} x_{a_1}(k+1) \\ x_{a_2}(k+1) \\ x_{a_3}(k+1) \\ \hline x_{g1}(k+1) \\ x_{g2}(k+1) \\ x_{g3}(k+1) \end{pmatrix} = \left[\begin{array}{c|c} A_a & 0_{3\times 3} \\ \hline 0_{3\times 3} & I_{3\times 3} \end{array}\right] \begin{pmatrix} x_{a_1}(k) \\ x_{a_2}(k) \\ x_{a_3}(k) \\ \hline x_{g1}(k) \\ x_{g2}(k) \\ x_{g3}(k) \end{pmatrix} + \left[\begin{array}{c} I_{3\times 3} \\ \hline -I_{3\times 3} \end{array}\right] \begin{pmatrix} u_1(k) - l_1(k) \\ u_2(k) - l_2(k) \\ u_3(k) - l_3(k) \end{pmatrix}, \tag{7}$$

which can be generalized for any N.

Loss of a center alters the structure of A_a. This change depends on whether the aircrafts are grounded or rerouted. We next derive the model for these two cases.

A. Approach 1: Rerouting

The first strategy mitigates the adverse effects of a center going down, is to reroute aircrafts, enroute to the lost center, to other centers. We ensure that the incoming traffic to the lost center is rerouted through centers which are its immediate neighbors in terms of connectivity. This is explained further with the help of a seven-center model shown in Fig. 1. The centers here are given by ZSE, ZOA, ZLA, ZLC, ZDV, ZAB, and ZMP. Assuming that ZLC goes down, we need to reroute the aircrafts through ZSE, ZOA, ZLA, ZDV, ZAB, and ZMP. Hence, the flow of aircrafts to and from these centers needs to be altered. This is achieved by first removing the columns and rows of A_a corresponding to the center that goes down, to obtain $\hat{A}_a \in \mathbb{R}^{(N-1)\times(N-1)}$, and then adding a perturbation matrix $\delta \in \mathbb{R}^{(N-1)\times(N-1)}$ to get $\tilde{A}_a(\delta) := \hat{A}_a + \delta$. The perturbation matrix δ determines which centers are to be used in rerouting. To ensure that only neighboring centers are used in the rerouting, the corresponding entries of δ have unknown terms, all the other terms in δ are set to zero. These unknown terms will be determined using a finite horizon optimization. For this example, with reference with Fig. 1, when ZLC goes down all the links with its neighboring centers are lost. This traffic is rerouted via its immediate centers, i.e., ZSE, ZOA, ZLA, ZDV, ZAB, and ZMP. Thus, the flow between these centers will have to be modified, using the existing connectivity. This is achieved by specifying δ to have the

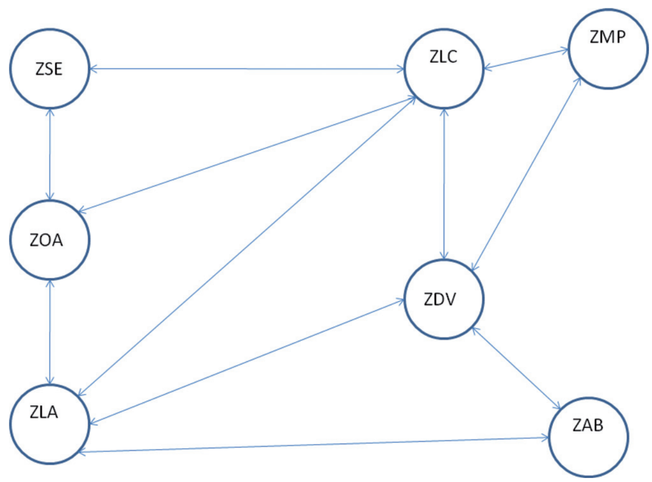

Fig. 1 Network with seven nodes.

following form:

$$\delta = \left(\begin{array}{c|cccccc} & ZSE & ZOA & ZLA & ZDV & ZAB & ZMP \\ \hline ZSE & 0 & 0 & 0 & 0 & 0 & 0 \\ ZOA & 0 & 0 & \delta_2 & 0 & 0 & 0 \\ ZLA & 0 & \delta_1 & 0 & \delta_4 & 0 & 0 \\ ZDV & 0 & 0 & \delta_3 & 0 & \delta_6 & 0 \\ ZAB & 0 & 0 & 0 & \delta_5 & 0 & 0 \\ ZMP & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right), \tag{8}$$

where $\delta_1, \ldots, \delta_6$ are unknowns and will be determined from optimization. In the event where more than one center is lost, the structure of δ can be appropriately defined using this approach. Note that, for every center lost, the dimension of the state vector $x := [x_a^{\mathrm{T}}\ x_g^{\mathrm{T}}]^{\mathrm{T}}$ reduces by two. Let us denote the state variables of the modified NAS by $\tilde{x}$. The modified equations of motion, with rerouting, can then be written as

$$\tilde{x}(k+1) = \left[\begin{array}{c|c} \tilde{A}_a(\delta) & 0_{(N-1)\times(N-1)} \\ \hline 0_{(N-1)\times(N-1)} & I_{(N-1)\times(N-1)} \end{array}\right] \tilde{x}(k) + \left[\begin{array}{c} I_{(N-1)\times(N-1)} \\ \hline -I_{(N-1)\times(N-1)} \end{array}\right] (\tilde{u}(k) - \tilde{l}(k)), \tag{9}$$

where $\tilde{u}$ and $\tilde{l}$ contain elements of u and l, except those corresponding to the center lost. For the following discussion, let us define

$$A_R(\delta) := \left[\begin{array}{c|c} \tilde{A}_a(\delta) & 0_{(N-1)\times(N-1)} \\ \hline 0_{(N-1)\times(N-1)} & I_{(N-1)\times(N-1)} \end{array}\right], \quad \text{and} \quad B_R := \left[\begin{array}{c} I_{(N-1)\times(N-1)} \\ \hline -I_{(N-1)\times(N-1)} \end{array}\right], \tag{10}$$

where the subscript R represents rerouting.

B. Approach 2: Ground-Delay

In this approach, we ground all the aircrafts, enroute to the lost center from other centers, to their respective grounds. With respect to the state-transition matrix $A_a = [A_{ij}^a]$, let us assume that the center i_0 goes down. Then all flights from centers $j = 1, \ldots, N$ and $j \neq i_0$, to center i_0 will be grounded at center j. This induces a delay in the flow of aircrafts in air. The flow out the ground states are influenced by takeoffs and landings. This modifies the dynamics of x_g for the modified system to:

$$\tilde{x}_g(k+1) = \tilde{x}_g(k) + [\mathrm{diag}(A^a_{i_0 j})_{j=1,\ldots,N;\, j\neq i_0}]\tilde{x}_a(k) + \tilde{l}(k) - \tilde{u}(k).$$

Therefore, the combined dynamics of the system can be written as

$$\tilde{x}(k+1) = A_G\tilde{x}(k) + B_G(\tilde{u}(k) - \tilde{l}(k)), \tag{11}$$

where

$$A_G := \left[\begin{array}{c|c} \hat{A}_a & 0_{(N-1)\times(N-1)} \\ \hline \mathrm{diag}(A^a_{i_0 j})_{j=1,\ldots,N;\, j\neq i_0} & I_{(N-1)\times(N-1)} \end{array}\right] \quad \text{and} \quad B_G := \left[\begin{array}{c} I_{(N-1)\times(N-1)} \\ \hline -I_{(N-1)\times(N-1)} \end{array}\right], \tag{12}$$

where the subscript G represents ground-delay. In the event, when several centers go down, i.e., $i_0 \in \mathcal{I} \subset \{1, 2, \ldots, N\}$, then A_G can be written as

$$A_G := \left[\begin{array}{c|c} \hat{A}_a & 0_{(N-1)\times(N-1)} \\ \hline \sum_{i_0\in\mathcal{I}} \mathrm{diag}(A^a_{i_0 j})_{j=1,\ldots,N;\, j\notin\mathcal{I}} & I_{(N-1)\times(N-1)} \end{array}\right]. \tag{13}$$

III. Problem Formulation

In this paper, we study three scenarios that may affect the performance of the NAS and investigate optimal mitigation schemes based on rerouting and ground-delay of flights. The three scienarios considered here are (a) loss of a center, (b) loss of a link between two center, and (c) isolation of a center over a time period. The optimal mitigation scheme is determined by solving a finite horizon problem with relevant contraints and dynamics. We assume that the NAS is in steady state before these events occur. Our objective in the first two scenarios is to minimize the deviation of the NAS from the steady-state values of the state vector. In the third scenario, our objective is to reduce the traffic flow to a given center to zero, over a given time period. It is assumed that these events occur at $k = 0$ and its effect manifests in the NAS over $k = 1, 2, \ldots$.

A. Loss of a Center

In this scenario, we are interested in minimizing the deviation from the steady-state value of $\tilde{x}$ in the event of a center loss, over a finite time horizon. That is, minimize $\|\tilde{x}(k) - \tilde{x}(0)\|_2$ for $k = 1, \ldots, r$, where r defines the length of the finite horizon. The cost function that achieves this objective can be written as

$$J = \sum_{j=1}^{r} (\tilde{x}(j) - \tilde{x}(0))^{\mathrm{T}}(\tilde{x}(j) - \tilde{x}(0)), \tag{14}$$

Depending on whether rerouting or ground-delay is used for mitigation, Eq. (9) or Eq. (11) can be use to rewrite Eq. (14). Let us denote $J_{\mathrm{R}}(\delta, \tilde{u}, \tilde{l})$ to be the cost function when rerouting is used and $J_{\mathrm{G}}(\tilde{u}, \tilde{l})$ to be the cost function when ground-delay is used. Note the dependency of J_{R} and J_{G} on the unknowns $\delta, \tilde{u}, \tilde{l}$. The value of $\tilde{x}$ at the rth time step can be written in terms of the initial condition $\tilde{x}(0)$, inputs $\tilde{u}, \tilde{l}$, and parameter δ as (shown here using Eq. (9)):

$$\tilde{x}(r) = A^r_{\mathrm{R}}\tilde{x}(0) + \begin{bmatrix} A^{r-1}_{\mathrm{R}} B_{\mathrm{R}} & A^{r-2}_{\mathrm{R}} B_{\mathrm{R}} & \cdots & B_{\mathrm{R}} \end{bmatrix} \begin{pmatrix} \tilde{u}(0) - \tilde{l}(0) \\ \tilde{u}(1) - \tilde{l}(1) \\ \vdots \\ \tilde{u}(r-1) - \tilde{l}(r-1) \end{pmatrix}. \tag{15}$$

Using Eq. (15) we can write J_{R} and J_{G} as

$$J_{\mathrm{R}} = L_{\mathrm{R}}(\delta) + M_{\mathrm{R}}(\delta)^{\mathrm{T}}\rho_{\mathrm{R}} + \rho^{\mathrm{T}}_{\mathrm{R}} N_{\mathrm{R}}(\delta)\rho_{\mathrm{R}}, \tag{16}$$

$$J_{\mathrm{G}} = L_{\mathrm{G}} + M^{\mathrm{T}}_{\mathrm{G}}\rho_{\mathrm{G}} + \rho^{\mathrm{T}}_{\mathrm{G}} N_{\mathrm{G}}\rho_{\mathrm{G}}, \tag{17}$$

where matrices $L_{\mathrm{R}}, L_{\mathrm{G}}, M_{\mathrm{R}}, M_{\mathrm{G}}, N_{\mathrm{R}}, N_{\mathrm{G}}$ are obtained by expanding Eq. (14) using Eq. (15). Note that Eq. (15) has to be rewritten in terms of $A_{\mathrm{G}}, B_{\mathrm{G}}$ when considering the ground-delay option. The unknowns, or the parameters of optimization, in the cost functions in Eqs. (16) and (17) are defined as:

$$\rho_{\mathrm{R}} = [\delta_1, \ldots, \delta_Q, \tilde{u}(0), \ldots, \tilde{u}(r-1), \tilde{l}(0), \ldots, \tilde{l}(r-1)]^{\mathrm{T}}, \tag{18}$$

$$\rho_{\mathrm{G}} = [\tilde{u}(0), \ldots, \tilde{u}(r-1), \tilde{l}(0), \ldots, \tilde{l}(r-1)]^{\mathrm{T}}, \tag{19}$$

where Q is the number of variables needed to redefine the state-transition matrix, determined from the connectivity of the lost center. In Eq. (16), note that the matrices L_{G}, M_{G} and N_{G} are constants, which makes J_{G} a quadratic function of ρ_{G}. Whereas the matrices L_{R}, M_{R}, and N_{R} in Eq. (17), are polyonomials in δ_i which makes J_{R} a polynomial function of ρ_{R}.

For the optimal solution to be meaningful we impose the following constraints on the problem:

C1: Preserve Markov structure (applicable only when rerouting is used):

$$\sum_{j=1}^{2N-2} A_{\mathrm{R}}(\delta)_{ij} = 1, \quad i = 1, 2, \ldots, 2N-2. \tag{20}$$

Preservation of the Markov structure ensures that the total number of aircrafts in the NAS are constant. This constraint is applicable only for rerouting because the state-transition matrix is modified in this case. For ground-delay, the state-transition matrix is Markov by definition.

C2: Positivity of δ, implying excess flow is added to the neighboring links (applicable only when re-routing is used):

$$\delta_i \geqslant 0, \quad i = 1, \ldots, Q. \tag{21}$$

Positivity of δ is necessary as we do not want to reduce the existing transitions between the modified links.

C3: For every center, at every time step, the number of takeoffs cannot be more than the number of aircrafts on the ground, and must be nonnegative:

$$0 \leqslant u_i(j) \leqslant x_{gi}(j), \quad j = 1, 2, \ldots, r, \ \ i = 1, 2, \ldots, N-1. \tag{22}$$

C4: For every center, at every time step, the number of landings cannot be more than number of aircrafts in air, and must be non-negative:

$$0 \leqslant l_i(j) \leqslant x_i(j), \quad i = 1, 2, \ldots, N-1, \quad \text{and} \quad j = 1, 2, \ldots, r. \tag{23}$$

C5: Number of aircrafts in air and on ground must be withing center capacity:

$$0 \leqslant x_i(j) \leqslant x_{\max}(i) \text{ and } 0 \leqslant x_{gi}(j) \leqslant x_{\mathrm{gmax}}(i), \quad i = 1, 2, \ldots, N-1, \ \ j = 1, 2, \ldots, r, \tag{24}$$

where $x_{\max}(i)$ and $x_{\mathrm{gmax}}(i)$ refers to the maximum air and ground capacity of ith center.

Note that the constraints C1–C5 are *linear* in the unknowns δ, $\tilde{u}$, and $\tilde{l}$. The optimization problem is, therefore, to minimize J_{R} or J_{G} with contraints C1–C5. If J_{G} is used as a cost function, then the problem is a constrained quadratic programming (QP) problem, which can be solved very efficiently, and will result in globally optimal solution. If J_{R} is used, then the problem is a nonlinear programming problem (NLPP) and the optimal solution will be local.

B. Loss of a Link Between Two Centers

In this case, the problem is similar to the center loss problem. This does not cause the number of centers to reduce, but sets $A_{ij} = 0$ in the state-transition matrix corresponding to the link that is lost. The control objective here is to minimize the deviation from the steady-state operation of the NAS before the link was lost. This can be achieved using a cost function that is similar to Eq. (14), but differs in number of terms to be added. It is given by Eq. (25)

$$J = \sum_{j=1}^{r} (x(j) - x(0))^{\mathrm{T}} (x(j) - x(0)), \tag{25}$$

Equations (26a–d) gives us the representation of the constraint equations when a particular link is removed. These constraint are same as C1–C5, except that they include all the N centers.

$$\sum_{j=1}^{2N} A(i, j) = 1, \quad i = 1, \ldots, 2N. \tag{26a}$$

$$0 \leqslant u_i(j) \leqslant x_{gi}(j), \quad j = 1, \ldots, r, \;\; i = 1, \ldots, N. \tag{26b}$$

$$0 \leqslant l_i(j) \leqslant x_i(j), \quad i = 1, \ldots, N, \;\; j = 1, 2, \ldots, r. \tag{26c}$$

$$0 \leqslant x_i(j) \leqslant x_{\max}(i) \text{ and } 0 \leqslant x_{gi}(j) \leqslant x_{\mathrm{gmax}}(i), \quad i = 1, \ldots, N, \;\; j = 1, \ldots, r. \tag{26d}$$

As in the previous case, we can use rerouting or ground-delay to minimize the cost function. The final form of the cost function, and whether we solve an NLPP or a QP, will depend on this choice.

C. Isolation of a Center in a Time Period

Here we consider the situation when a given center has to be isolated within a given time horizon T. This may be due to an imminent severe weather condition, or security-related reason. Here our aim is to the reduce air traffic, under center i_0, to zero over a given time period T. The control objective here is to minimize the deviation from the steady-state operation of the NAS while and after the center i_0 is fazed out. This is achieved by defining a cost function as

$$J = \sum_{j=1}^{r} \left\{ \sum_{i=1, i \neq i_0}^{N} [(x_{a_i}(j) - x_{a_i}(0))^2 + (x_{g_i}(j) - x_{g_i}(0))^2] + W_j x_{i_0}(j) \right\}, \tag{27}$$

where $r < T$ and $W_1, W_2, \ldots, W_r$ are the weights in increasing order, i.e.,

$$W_r \geqslant W_{r-1} \geqslant W_{r-2} \geqslant \cdots W_2 \geqslant W_1. \tag{28}$$

The increasing weight functions ensure that the state variable related to the center i_0 is convergent in nature and a sequence of optimizations will result in reducing $x_{i_0}(k)$ to zero. Clearly, the choice of the weight functions will determine the rate of convergence. The constraints here are exactly the same as those in Eq. (26). Once again, we can use rerouting or ground-delay to minimize the cost function, and the nature of the optimization problem will depend on this.

IV. Results

A. A Simple Example

We first deal with the seven-center model shown in Fig. 1. The δ matrix is formed as explained in Section IIA. For example, with reference to Fig. 1, if we remove ZLA and consider rerouting as an option, the δ matrix is given by

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 \\ 0 & \delta_1 & 0 & \delta_4 & 0 & 0 \\ 0 & 0 & \delta_3 & 0 & \delta_6 & 0 \\ 0 & 0 & 0 & \delta_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{29}$$

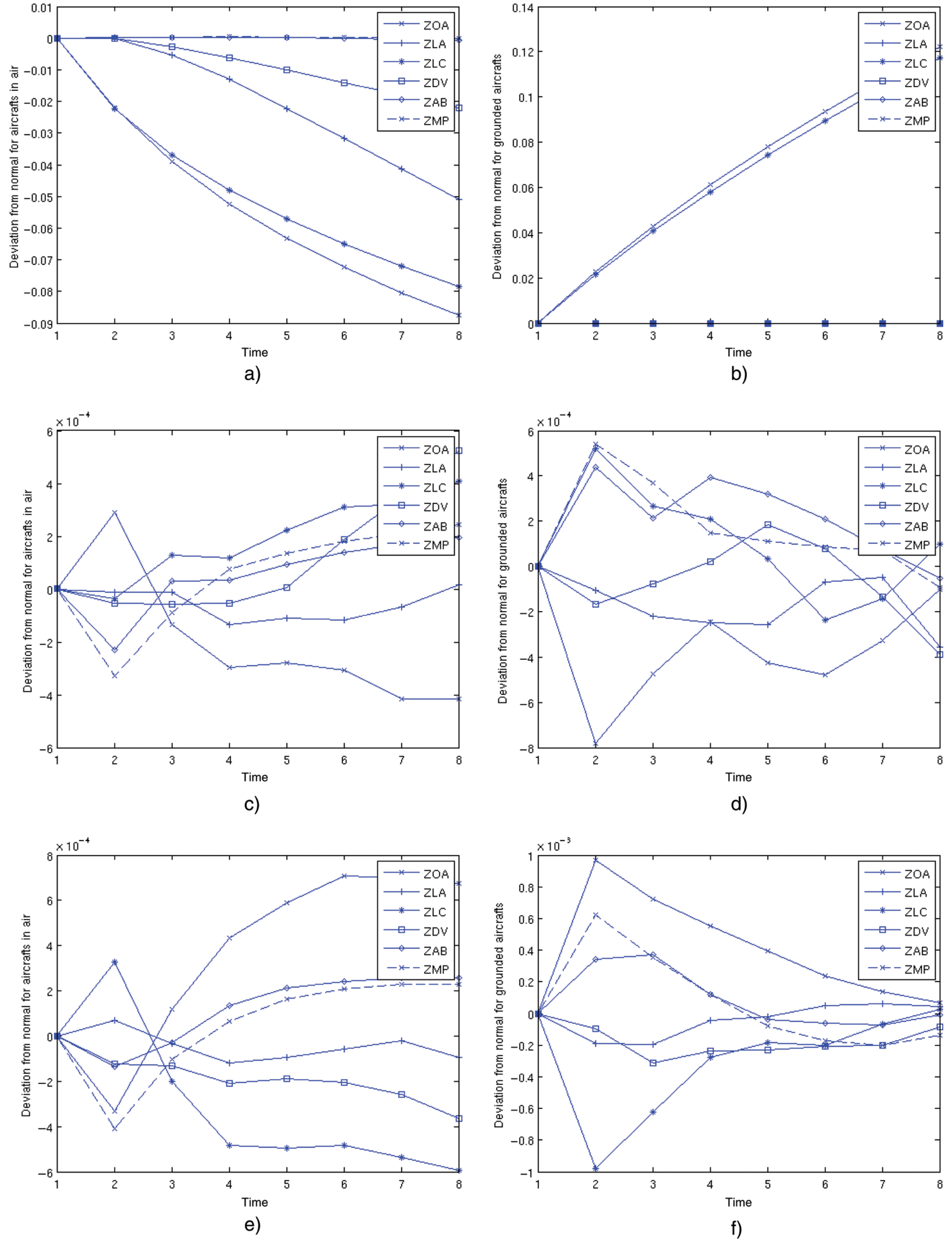

Fig. 2 ZSE down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

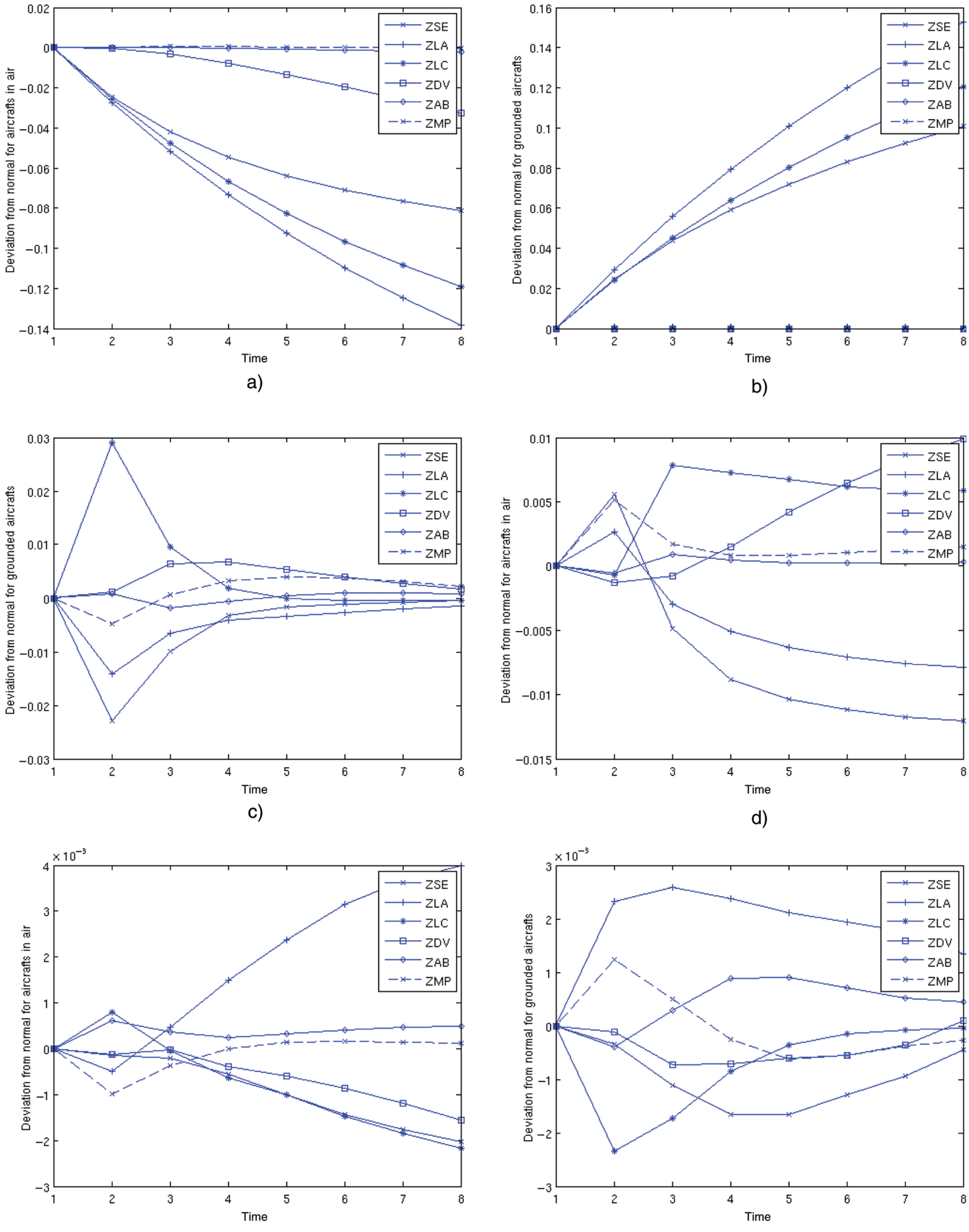

Fig. 3 ZOA down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

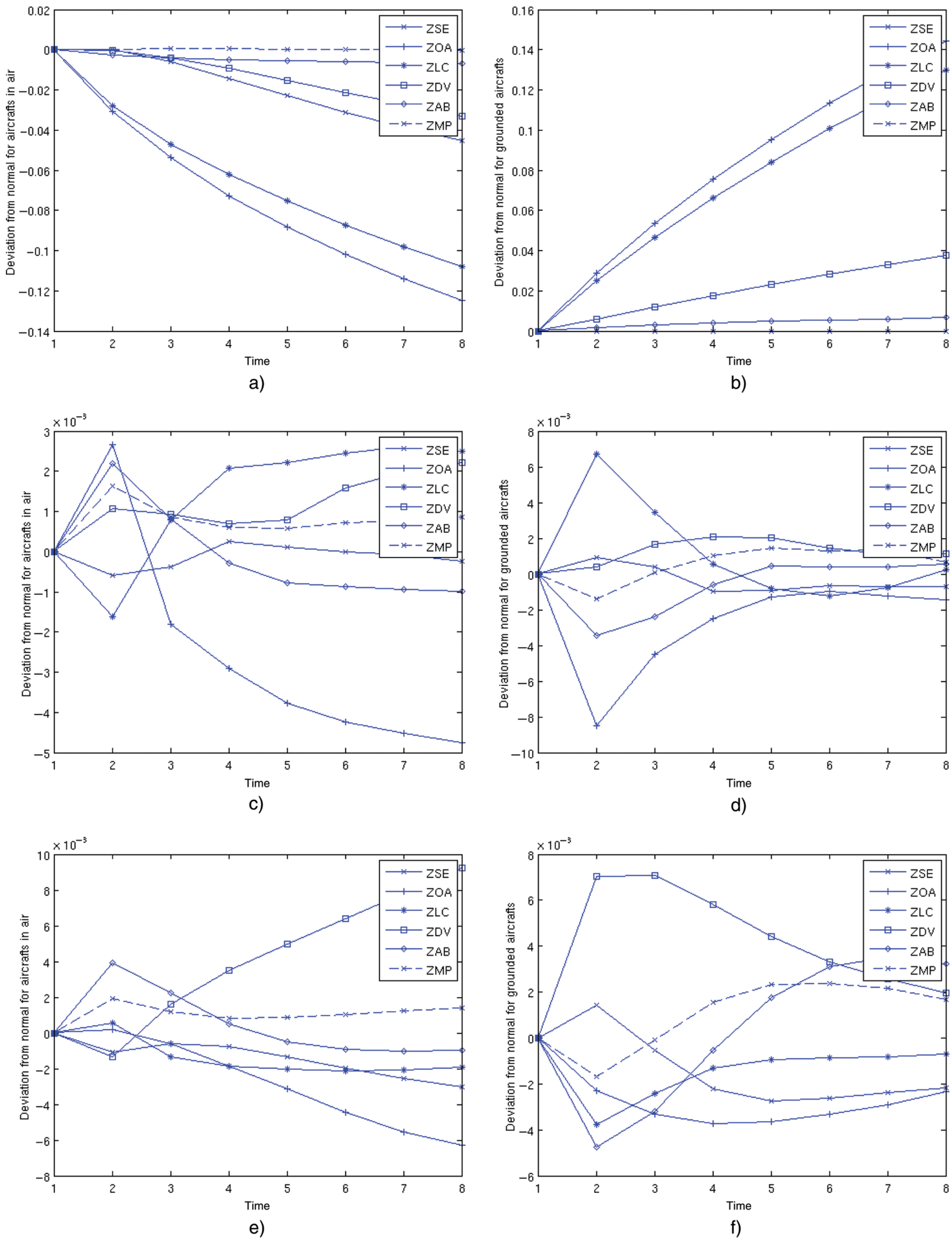

Fig. 4 ZLA down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

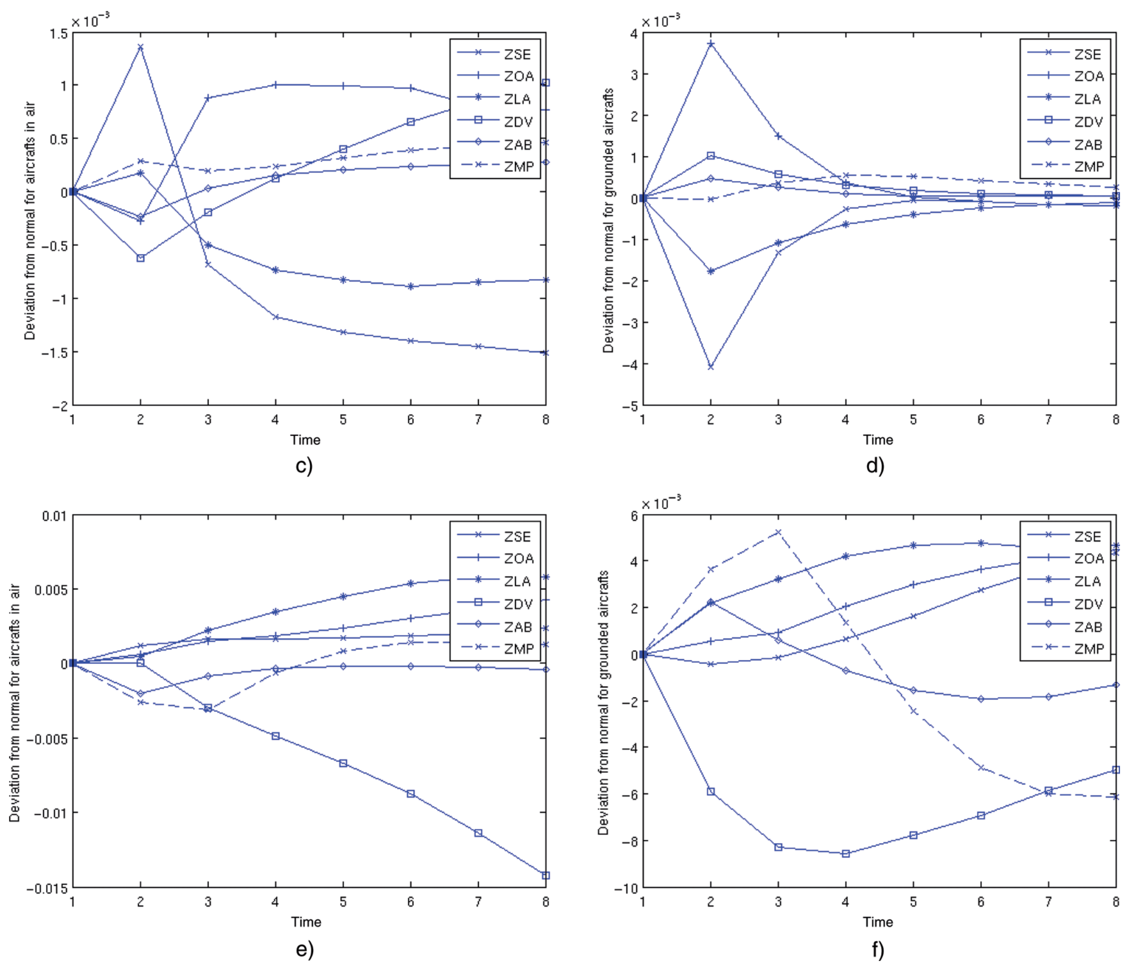

Fig. 5 ZLC down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

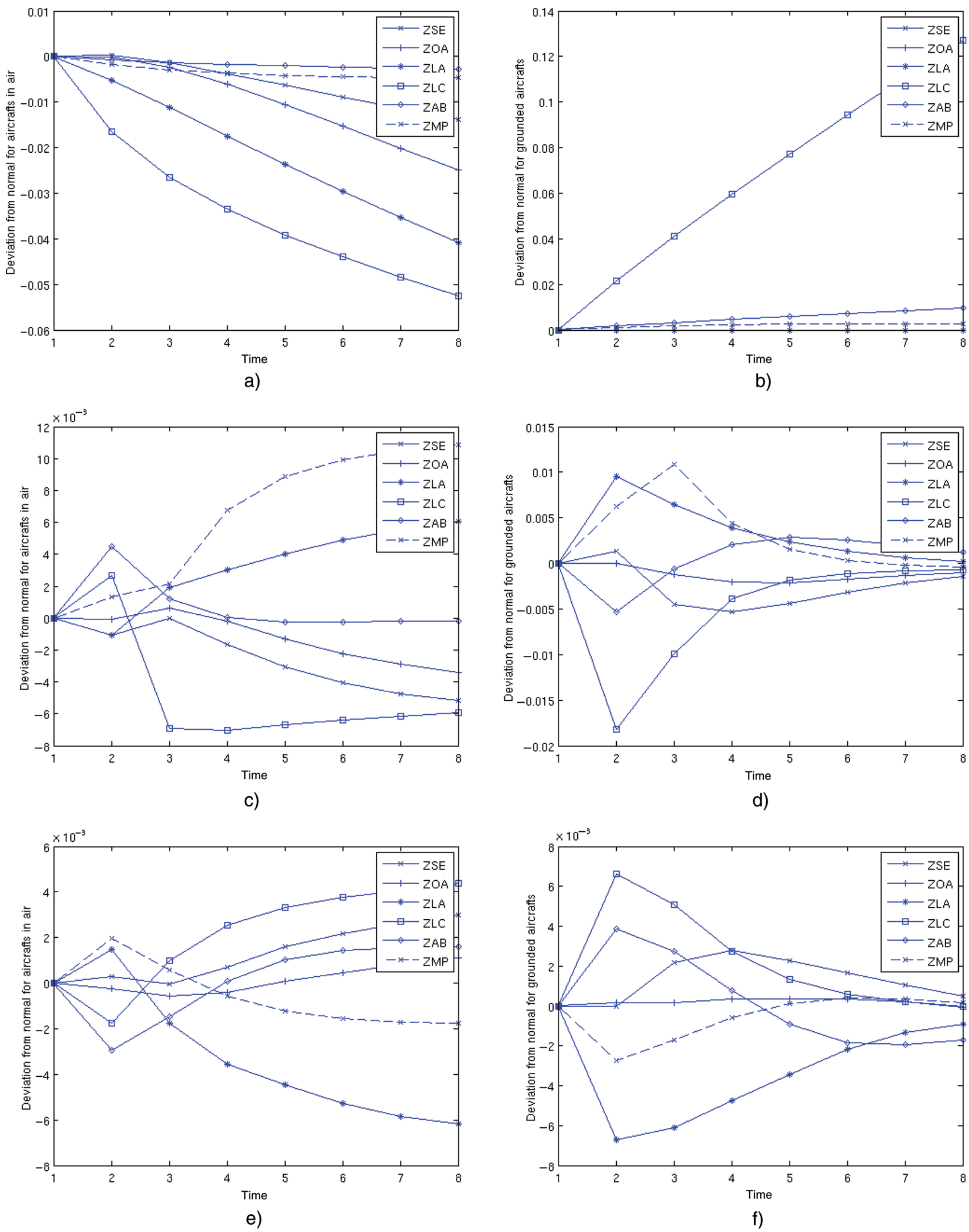

Fig. 6 ZDV down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

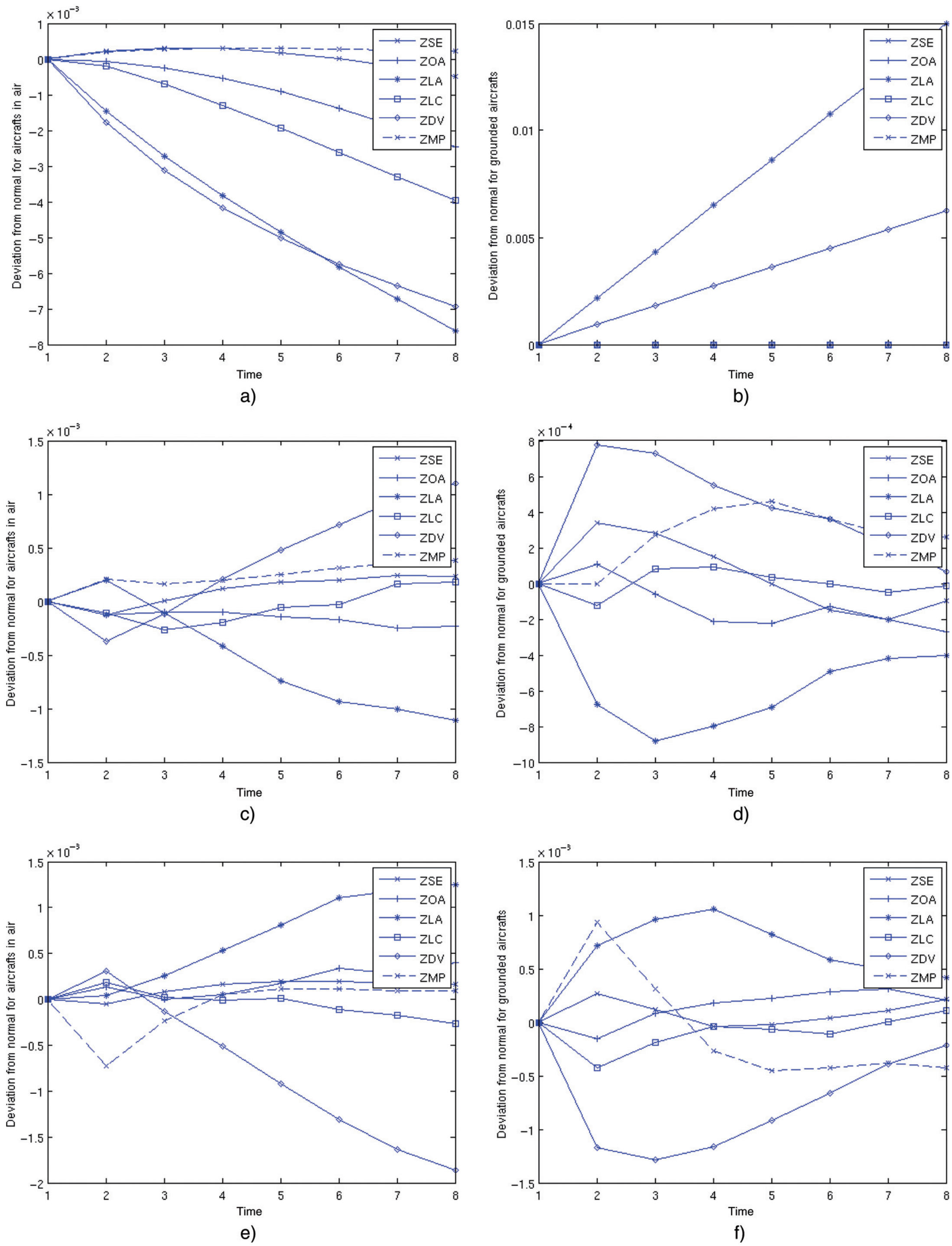

Fig. 7 ZAB down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

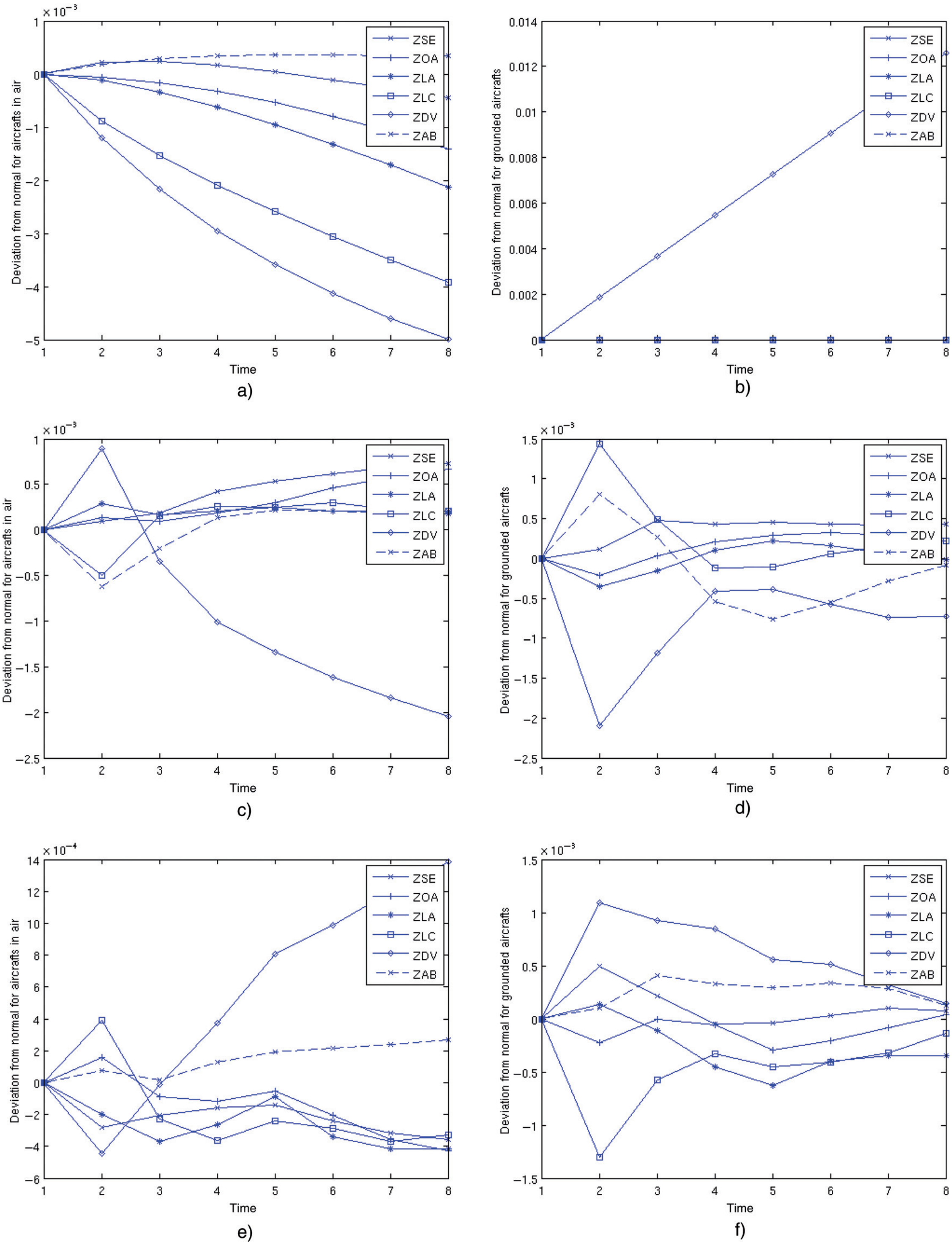

Fig. 8 ZMP down: a) deviations in x_a (nominal); b) deviations in x_g (nominal); c) deviations in x_a (rerouting); d) deviations in x_g (rerouting); e) deviations in x_a (ground-delay); and f) deviations in x_g (ground-delay).

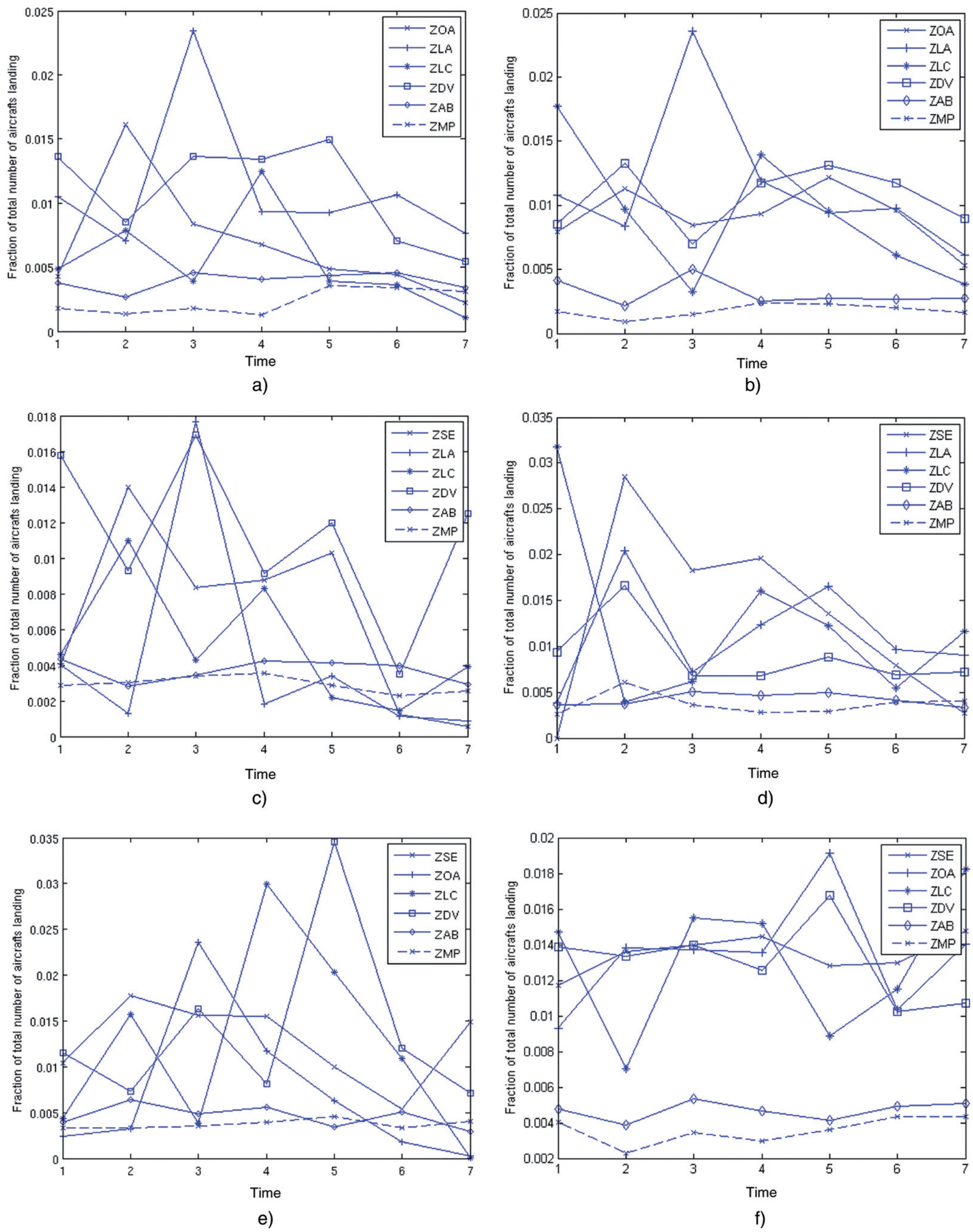

Fig. 9 Comparison of landing trajectories in case of ground-delay and rerouting, for various center taken out: a) landing trajectories for ground-delay — center ZSE out; b) landing trajectories for rerouting — center ZSE out; c) landing trajectories for ground-delay — center ZOA out; d) landing trajectories for rerouting — center ZOA out; e) landing trajectories for ground-delay — center ZLA out; f) landing trajectories for rerouting — center ZLA out; g) landing trajectories for ground-delay — center ZLC out; h) landing trajectories for rerouting — center ZLC out; i) landing trajectories for ground-delay — center ZDV out; j) landing trajectories for rerouting — center ZDV out; k) landing trajectories for ground-delay — center ZAB out; l) landing trajectories for rerouting — center ZAB out; m) landing trajectories for ground-delay — center ZMP out; and n) landing trajectories for rerouting — center ZMP out.

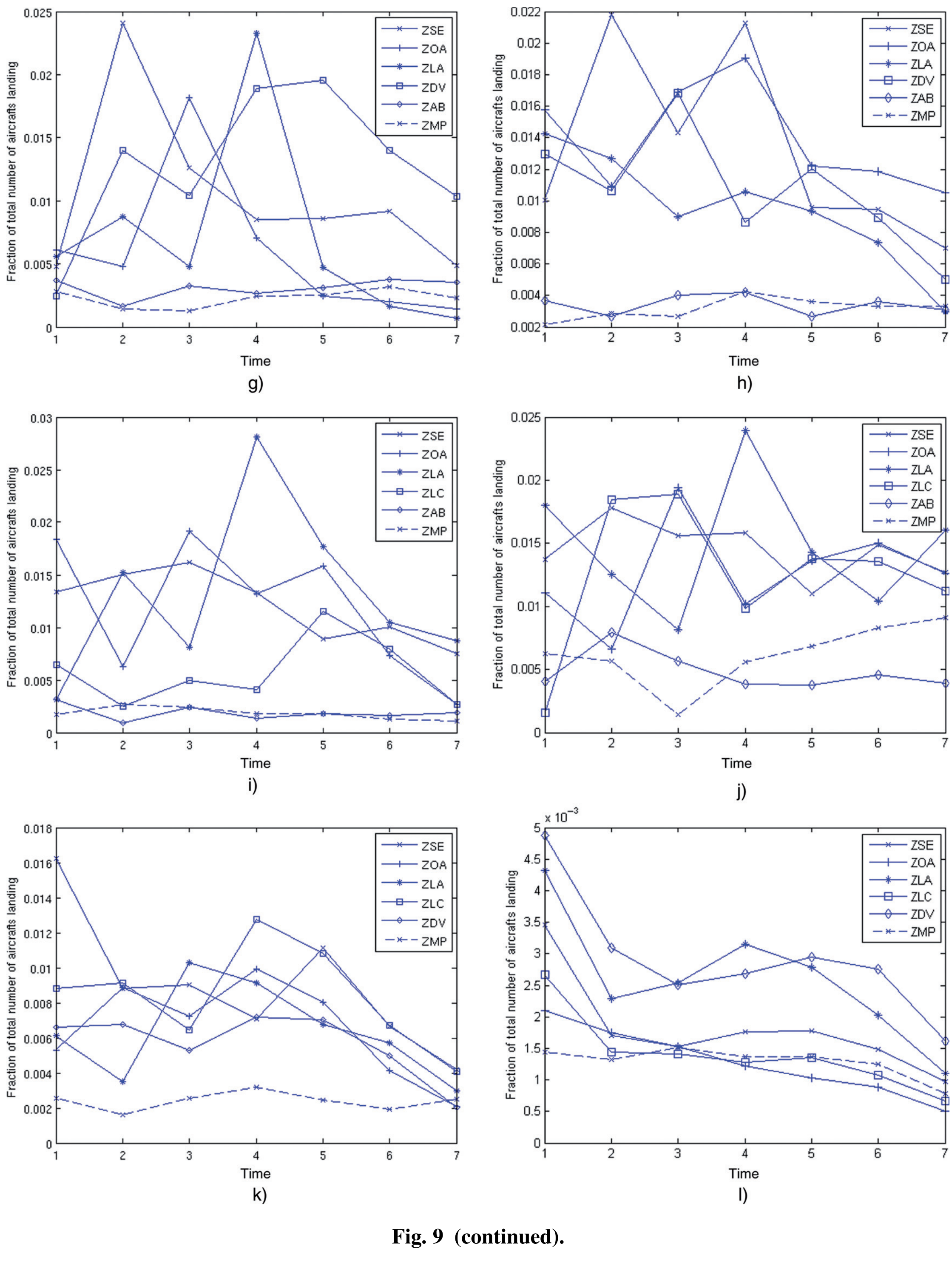

Fig. 9 (continued).

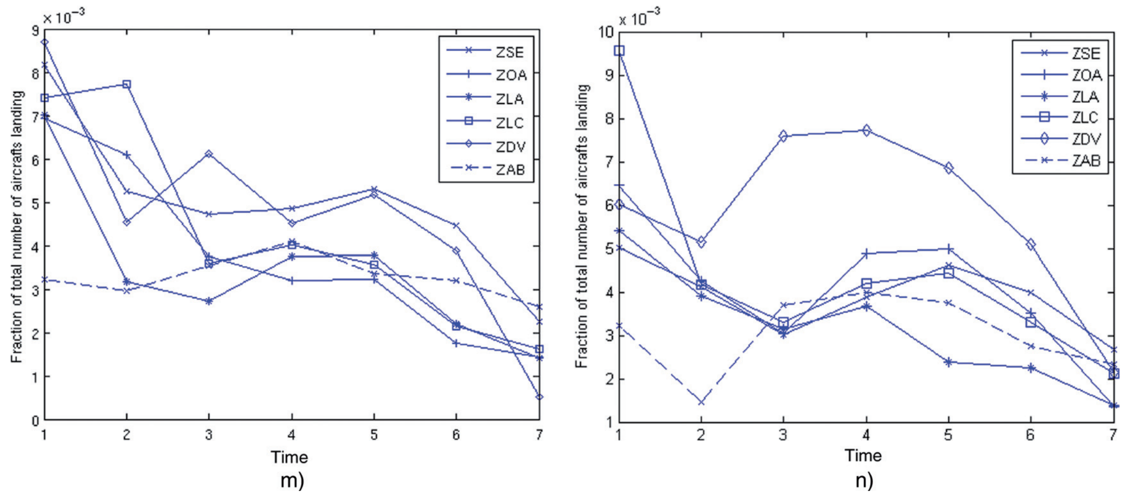

Fig. 9 (continued).

Then A_{R} is

$$A_{\mathrm{R}} = \begin{pmatrix} 0.5788 & 0.1103 & 0.1101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2106 & 0.6588 & 0.1101+\delta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2106 & 0.1103+\delta_1 & 0.5596 & 0.1609+\delta_4 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1101+\delta_3 & 0.7588 & 0.21+\delta_6 & 0.22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01+\delta_5 & 0.61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.02 & 0 & 0.63 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{30}$$

If ground-delay is used, then

$$A_{\mathrm{G}} = \begin{pmatrix} 0.5788 & 0.1103 & 0.1101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2106 & 0.6588 & 0.1101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2106 & 0.1103 & 0.5596 & 0.1609 & 0 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1101 & 0.7588 & 0.21 & 0.22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0.61 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.02 & 0 & 0.63 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1206 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1101 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0503 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.18 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{31}$$

With these two models, we perform the optimization over seven time steps, with cost function:

$$J = \sum_{i=1}^{6} \sum_{j=1}^{7} (x_{ij} - x_{0i})^2 + \sum_{i=1}^{6} \sum_{j=1}^{7} (x_{gij} - x_{g0i})^2 \tag{32}$$

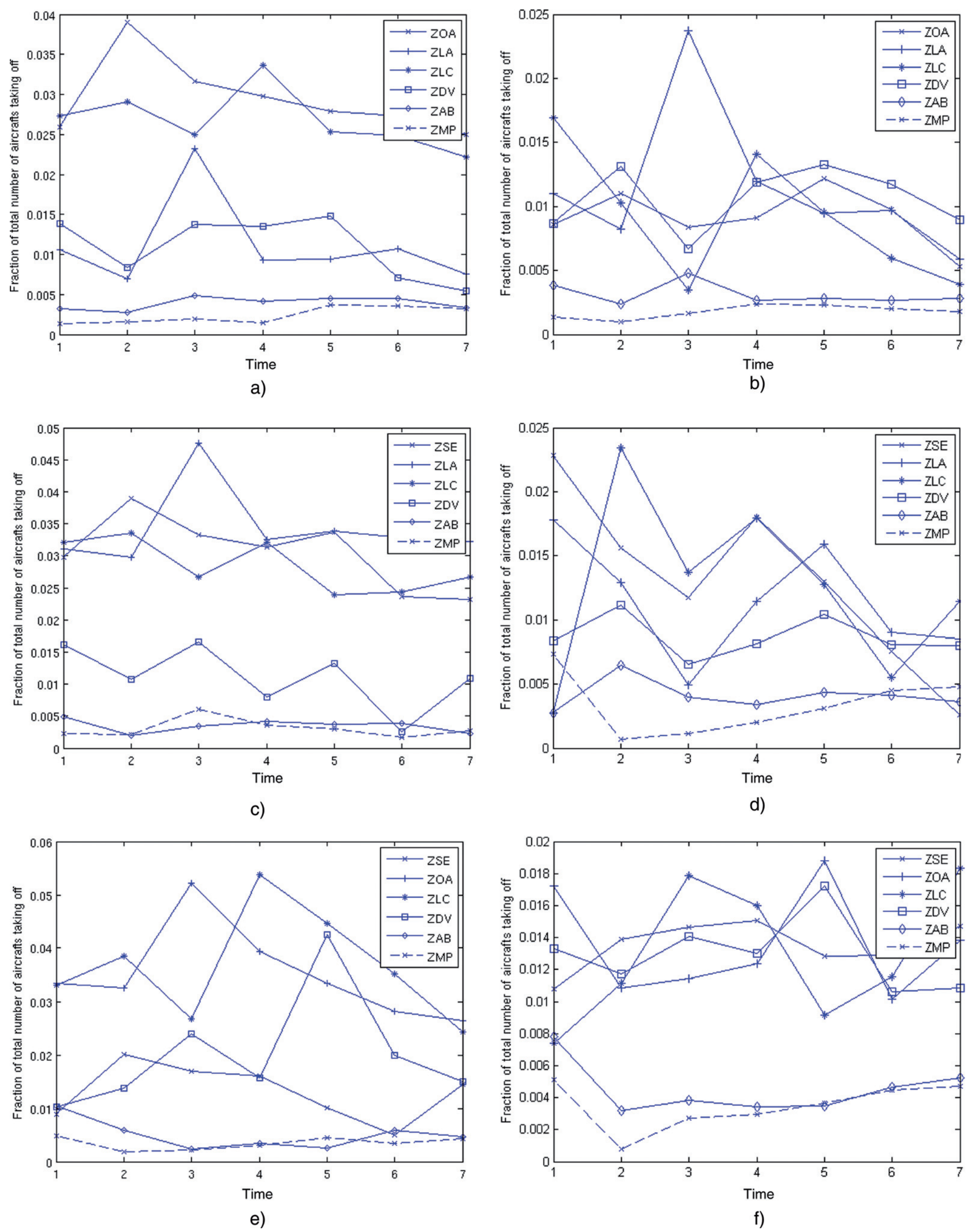

Fig. 10 Comparison of takeoff trajectories in case of ground-delay and rerouting, for various center taken out: a) takeoff trajectories for ground-delay — center ZSE out; b) takeoff trajectories for rerouting — center ZSE out; c) takeoff trajectories for ground-delay — center ZOA out; d) takeoff trajectories for rerouting — center ZOA out; e) takeoff trajectories for ground-delay — center ZLA out; f) takeoff trajectories for rerouting — center ZLA out; g) takeoff trajectories for ground-delay — center ZLC out; h) takeoff trajectories for rerouting — center ZLC out; i) takeoff trajectories for ground-delay — center ZDV out; j) takeoff trajectories for rerouting — center ZDV out; k) takeoff trajectories for ground-delay — center ZAB out; l) takeoff trajectories for rerouting — center ZAB out; m) takeoff trajectories for ground-delay — center ZMP out; n) takeoff trajectories for rerouting — center ZMP out.

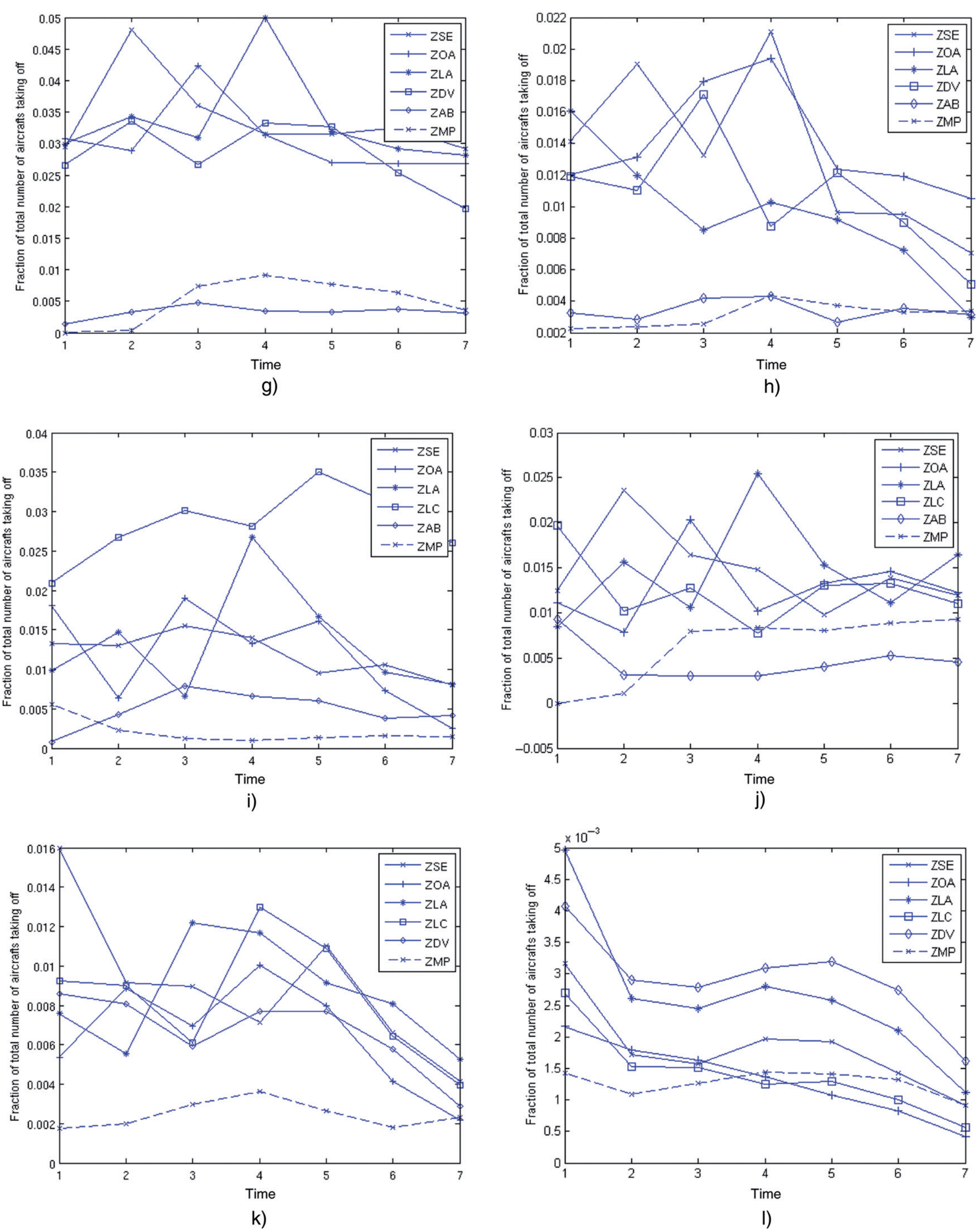

Fig. 10 (continued).

Time history of the deviation of the states from initial values is shown in Figs. 2 to 8. The initial value of the states are the steady-state values before the loss of a center. These figures represent the evolution of the NAS when each center is taken out in the seven-state model. The “nominal” case corresponds to the event when a center is lost and all the inbound traffic to that center are grounded at the orginating center. As we observe from the plots, this causes the number of aircrafts in air, for most of the centers, to decrease. Also, the number of aircrafts on ground

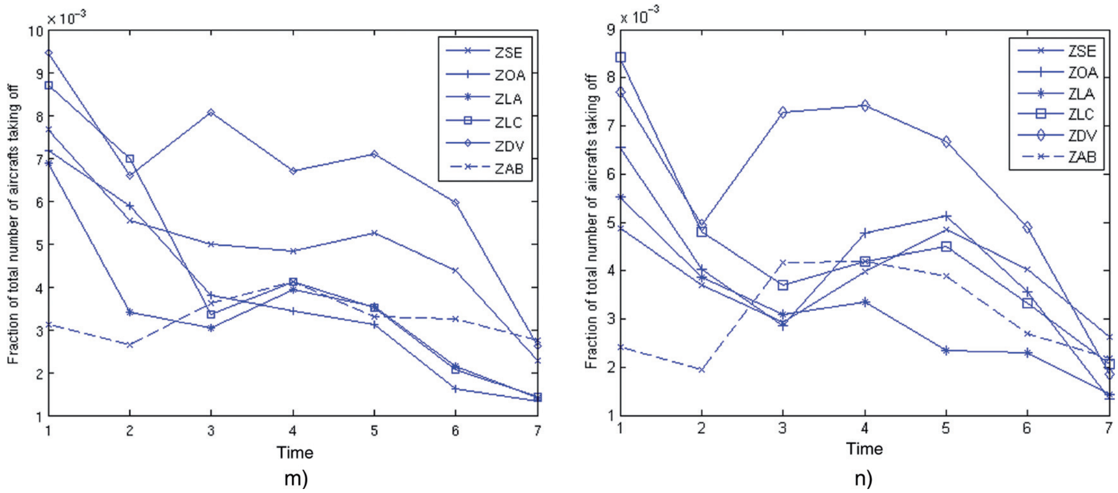

Fig. 10 (continued).

increases. Thus, this causes a significant deviation from the steady-state flow before the center being taken out. It is also interesting to note that for some centers, the deviation for both aircraft on ground and air are zero. This is because the aircraft flow in some centers are not affected by this event. This is governed by the relative connectivity of the centers in the NAS. When we apply optimal rerouting and ground-delay, we observe that the deviation from steady-state flow is quite small, thus highlighting the benefit of this strategy. The difference between the nominal strategy and ground-delay strategy is that in the nomimal case there is no control in terms of takeoffs and landings. In ground-delay, the control variables u and l regulate the airflow in NAS to be close to the steady-state flow.

Thus both these methods restore the NAS performance satisfactorily and perform equally well. From the control trajectories in Figs. 9 and 10 it is difficult to assess the impact of these strategies on the control trajectories. But when we consider the average behavior of these trajectories then some trends become clear. In Fig. 11 we plot the average control trajectories. The averages are computed over time (temporal averages) for every center; and over center (spatial averages), for every time step. Figure 11 shows the behavior of these average trajectories for the two strategies, for every case. We observe that the spatial and temporal average landing trajectories are similar for every case averages. But the number of takeoffs are, in general, higher for ground-delay. This is more apparent when we compute the spatial and temporal averages by combining all the case studies. Figure 12 shows these trajectories. However, these plots show minor differences between the two strategies in the seven-center model and we can conclusively state that there is no significant difference in performance between the two methods.

We next study the 20-center model, a graphical representation of which is shown in Fig. 13. We use the discretized version of the state equations given by Eqs. (1) and (2). The initial conditions were taken to be the greatest integer values for the steady-state number of aircrafts of the model without control. This was done mainly to have a integer value for number of aircrafts which are given in Table 1.

We ran the model, assuming a center is lost at a particular time of the day. The state transition matrix was taken from the literature [8] which denotes aircraft flows in the NAS from 0 through 1 Universal Coordinated Time (UTC), which is 7 p.m. Eastern Standard Time (EST) to 8 p.m. EST. The traffic in this 1-h period of time is heavy [8]. The state transition matrix is given in Table 2.

The rows in Table 2 refers to the destinations of aircrafts and the columns provide their originating centers. The figures in Table 2 refers to the fraction of aircrafts in transit. The columns do not add up to one, so they are normalized to represent the transition probabilities.

We test the model by removing one center at a time and optimizing the cost function given by Eq. (14) with $N = 20$ and $r = 7$. As in the case of the seven-state model we observe that the both these strategies are able to minimize the deviation of NAS from steady-state performance. Also, as before we did not observe a significant difference in the behavior of the controlled NAS when ground-delay or rerouting was used. Instead of plotting the

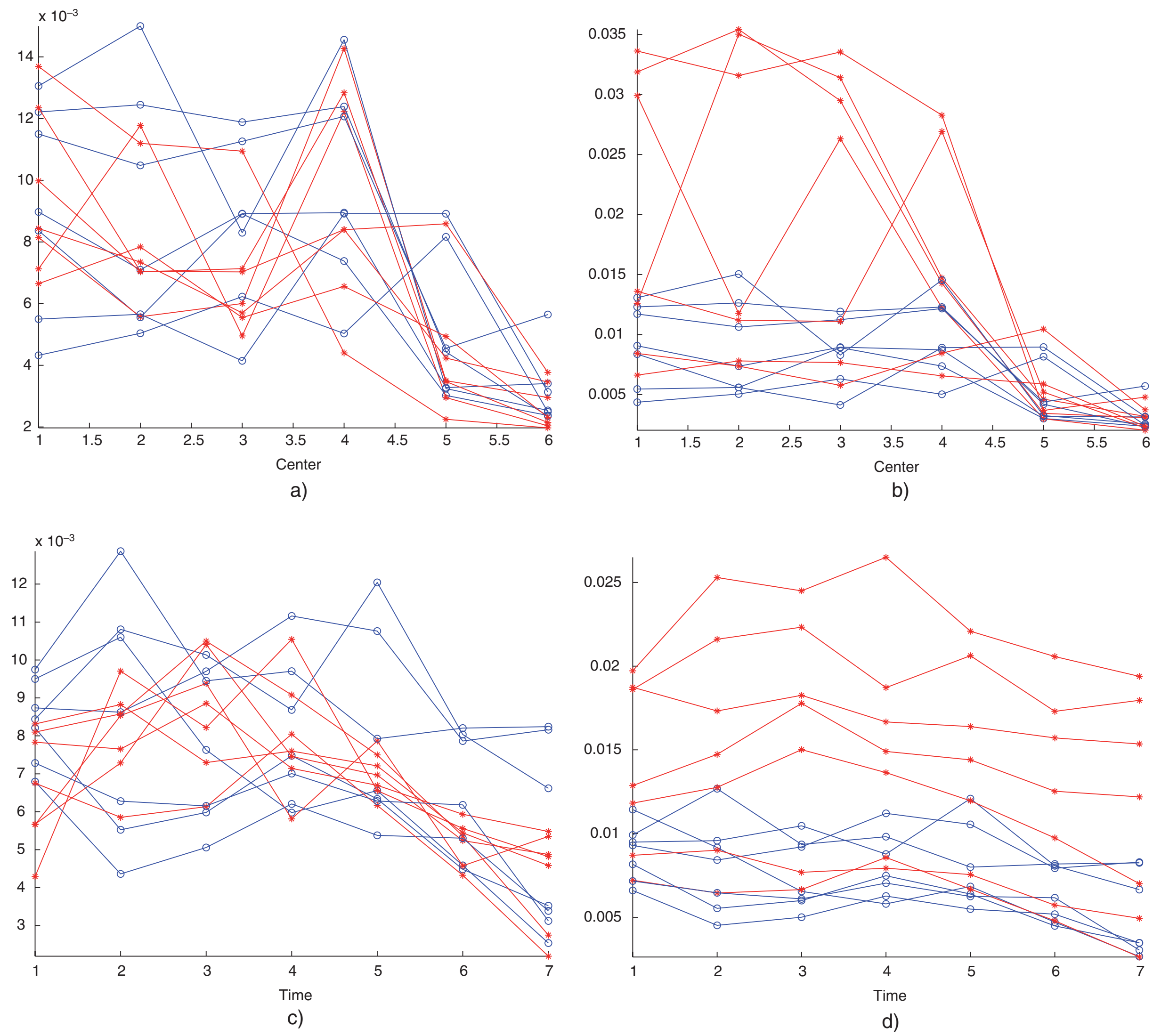

Fig. 11 Time and center averages of control trajectories for every case. Asterisk denote ground-delay, circles denote rerouting: a) time average of landing trajectories for each center; b) time average of takeoff trajectories for each center; c) center average of landing trajectories for each center; and d) center average of takeoff trajectories for each center.

trajectories for all the cases, we summarize the results by showing the percentage difference in number of aircrafts in air due to these two strategies. These are given in Table 3. Here the columns correspond to the center that is lost and the rows correspond to the percentage change in aircrafts in air, due a center being lost. The numbering of the rows corresponds to the sequence of centers in the columns, skipping the center that is lost. Note that the columns for ZMA and ZKC in Table 3 have been left blank. If we refer to Fig. 13, ZMA has a single link that connects it to ZJX, removing which completely disconnects ZMA from rest of the NAS. Hence, rerouting cannot be observed in that scenario. ZKC is the most connected center in the NAS. Removing it results in nine links getting cut, which leads to excessive number of variables in our workspace, thus making the rerouting algorithm computationally difficult to solve.

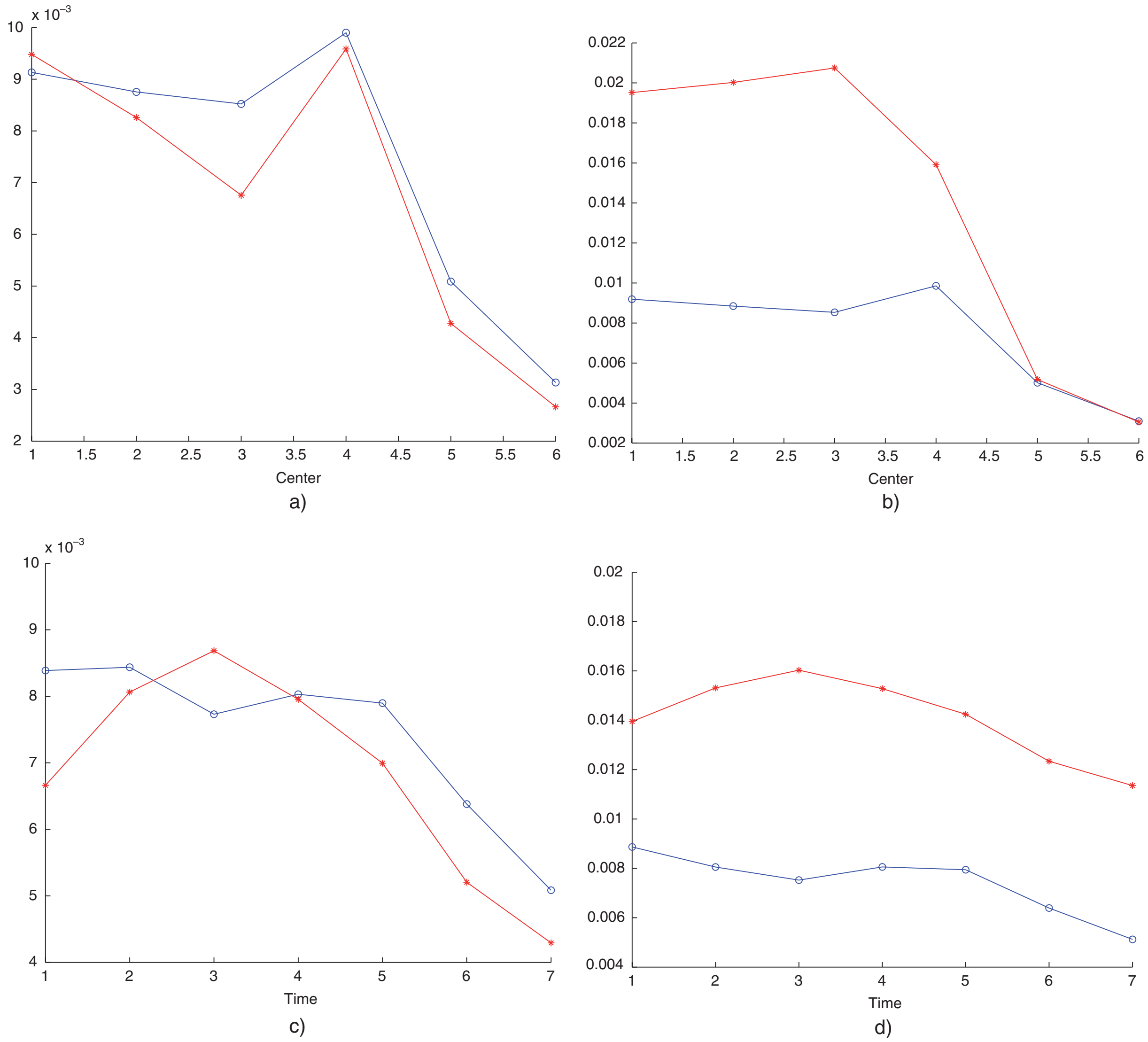

Fig. 12 Time and center averages of control trajectories, averaged over every case. Asterisk denote ground-delay, circles denote rerouting: a) time average of landing trajectories for each center; b) time average of takeoff trajectories for each center; c) center average of landing trajectories for each center; and d) center average of takeoff trajectories for each center.

In Table 3, we see that most of the differences are in the order of 1%, thus establishing the equivalence of these two methods. However, there are some entries with differences of almost 20%. We tried to correlate this anomaly with the degree and the algebraic connectivity of the center. We expected when centers with strong connectivity went down, rerouting will prove to be a better option. However, there were no conclusive evidence from the simulation plots. To analyze this further, we looked at the second vector of the Laplacian for the 20-center model. Figure 14 shows the second vector when we take out each center one at a time. We observe that, based on the dynamics of the NAS we have used, there is no clear decoupling or weak coupling in the dynamics. If there are weakly coupled subgraphs in the model, due to a center loss, then we expect to observe differences in the performance of the two strategies. It also turns out that the system in Eqs. (9) and (11) is controllable for all the cases. Hence, both the strategies work equally well. Additionally, Table 2 shows the diagonal dominance of the state-transition matrix, which indicates the

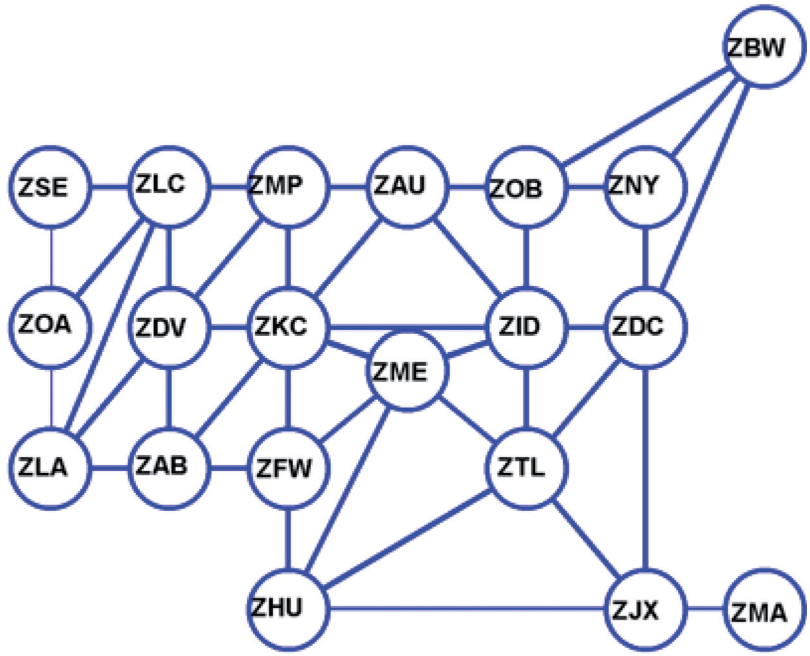

Fig. 13 Twenty center air-traffic control structure.

Table 1 Initial number of aircrafts

Centers	ZSE	ZOA	ZLA	ZLC	ZDV	ZAB	ZMP	ZKC	ZFW	ZHU	ZAU	ZME	ZOB	ZID	ZTL	ZNY	ZDC	ZJX	ZBW	ZMA
Number of aircrafts	382	352	551	447	147	214	152	19	77	75	29	21	35	27	73	55	85	52	158	39

weak influence of a given center on the rest of the NAS. For this reason, lost a center does not alter the dynamics of NAS significantly. For this reason, the control trajectories all have similar trend for all the cases considered.

V. Conclusion and Scope for Future Work

In this paper we demonstrated the effectiveness of using ground-delays and rerouting as strategies to mitigate the effect of a center going down in the NAS. We formulated a finite horizon-based optimal control strategy for both ground-delay and rerouting and presented the results for a seven-center model and a 20-center model of the NAS. The performance objective for the optimal control was to minimize the deviation of the number of aircrafts in air in the event a center goes down. The deviation was computed with respect to the steady-state flow rate in the NAS, before the loss of a center. The simulation plots show that both these methods are equally effective in restoring the performance of the NAS. Average case behavior indicate that ground-delay requires more landings to achieve the same performance, but this difference from rerouting is not significant to make this claim conclusively. However, in terms of computational point of view, ground-delay-based optimization is significantly simpler to solve, as it translates to a quadratic programming problem. The rerouting algorithm results in a nonlinear programming problem, which are in general difficult to solve.

In future work, we wish to apply time-varying model of the NAS in our analysis. The dynamics of NAS assumed here is kept constant at every time step. In reality, the dynamics changes and is an linear time invariant system. These models are not available in the literature in general. Therefore, we plan to use FACET [9] to extract appropriate Markov models from simulation data.

Table 2 State transition matrix from zero through one UTC

	zse	zoa	zla	zlc	zdv	zab	zmp	zkc	zfw	zhu	zau	zme	zob	zid	ztl	zny	zdc	zjx	zbw	zma
zse	0.9	0.01	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zoa	0.01	0.91	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zla	0	0.02	0.89	0.01	0.01	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zlc	0.01	0.01	0.01	0.93	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
zdv	0	0	0	0.01	0.92	0.01	0.02	0.02	0	0	0	0	0	0	0	0	0	0	0	0
zab	0	0	0.01	0	0.01	0.91	0	0	0.02	0	0	0	0	0	0	0	0	0	0	0
zmp	0	0	0	0	0.02	0	0.93	0.01	0	0	0.04	0	0.01	0	0	0	0	0	0	0
zkc	0	0	0	0	0	0	0	0.89	0.01	0	0.01	0.01	0	0.01	0	0	0	0	0	0
zfw	0	0	0	0	0	0.01	0	0.01	0.88	0.02	0	0.04	0	0	0	0	0	0	0	0
zhu	0	0	0	0	0	0	0	0	0.02	0.91	0	0.01	0	0	0	0	0	0.01	0	0
zau	0	0	0	0	0	0	0.01	0.02	0	0	0.86	0	0.02	0.02	0	0	0	0	0	0
zme	0	0	0	0	0	0	0	0.01	0.01	0	0	0.87	0	0.02	0.01	0	0	0	0	0
zob	0	0	0	0	0	0	0	0	0	0	0.04	0	0.83	0.03	0	0.02	0.01	0	0	0
zid	0	0	0	0	0	0	0	0.01	0	0	0.02	0.02	0.03	0.86	0.01	0	0	0	0	0
ztl	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0.02	0.88	0	0.02	0.03	0	0
zny	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0.77	0.03	0	0.03	0
zdc	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.02	0.05	0.87	0.02	0	0
zjx	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0.02	0	0.01	0.87	0	0.05
zbw	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0	0	0.08	0	0	0.88	0
zma	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.04	0	0.86

Table 3 The percentage difference in number of aircrafts, at final time, with rerouting and ground-delay

Center	ZSE	ZOA	ZLA	ZLC	ZDV	ZAB	ZMP	ZKC	ZFW	ZHU	ZAU	ZME	ZOB	ZID	ZTL	ZNY	ZDC	ZJX	ZBW	ZMA
1st Center	0.6544	−0.5429	0.009	0.5685	0.1129	−0.0616	0.205		−0.0723	0.0004	−0.0595	−0.024	0.0371	−0.0023	0.057	0.0683	0.0539	−0.0517	−0.1045	
2nd Center	0.0422	0.9663	−1.2167	0.8542	−0.7653	0.1159	−0.0606		−0.0838	0.031	−0.1527	0.0635	0.0023	−0.0332	−0.0169	−0.3072	0.0569	0.059	−0.179	
3rd Center	−0.48	−0.6042	1.179	−0.902	1.183	−0.7334	−0.1498		−0.0449	0.0016	−0.141	0.1481	−0.0226	−0.1043	−0.0772	−0.0144	−0.0516	0.1284	−0.0625	
4th Center	0.1766	0.2396	−2.8695	1.7882	−0.534	0.1757	0.0391		−0.0469	0.0087	−0.1902	0.1099	−0.0736	−0.1818	−0.1979	−0.4069	0.106	0.0966	0.1525	
5th Center	0.1393	−0.2654	1.9159	−0.0355	−0.8872	2.4559	−0.2543		0.0539	0.002	0.0933	0.4565	−0.0564	0.1341	0.9729	−0.4789	−0.0292	0.2698	0.1496	
6th Center	−0.0217	0.4062	−0.2039	2.1032	0.7835	0.8858	0.1167		−3.8946	−0.1335	−0.0582	0.2091	−0.2857	−0.4016	−0.0391	−0.1136	−0.0687	0.0027	0.2909	
7th Center	−0.087	−1.2374	0.0356	−2.7459	15.1733	20.4625	14.9717		0.2312	0.0159	−3.1899	0.2816	−0.2964	0.3868	−0.6658	0.3416	0.1316	0.3768	0.2193	
8th Center	−0.1317	−0.1382	−0.1633	0.663	−4.0895	−7.3505	−0.0306		12.3256	−0.4817	12.008	9.9559	−0.0443	1.8258	0.4301	0.5283	−0.5893	−1.5109	−0.9237	
9th Center	−0.0775	−1.522	−0.772	1.3425	−2.4333	−0.6323	−0.8389		−5.4231	−6.6258	−0.232	−2.9279	−0.2363	−0.6503	−0.0049	0.2009	0.3014	−0.3255	0.4627	
10th Center	0.2628	0.284	−0.4093	−1.9019	1.9063	1.1055	−12.9895		−0.4279	0.1041	0.7593	0.893	−0.0592	−1.0475	1.1784	1.4952	0.1548	−0.8324	−0.5682	
11th Center	−1.0918	1.6151	1.3244	1.3348	1.5175	1.535	1.1709		11.9922	21.553	0.3981	2.4961	−4.0659	6.3032	−2.5414	−0.1772	0.4302	−0.6805	0.0355	
12th Center	0.4613	0.8606	−0.6829	2.8862	−3.4803	−0.7946	−6.0638		−0.5903	−0.2015	−2.4243	1.6594	1.9268	1.2265	17.5946	1.3717	1.3382	0.0995	0.8319	
13th Center	0.0301	0.2815	−0.0702	−0.2441	−4.5407	−0.5028	−0.0748		−0.6225	0.3243	4.7614	0.3755	6.5813	−7.8322	−0.4468	6.8239	−4.4125	−0.103	14.7195	
14th Center	−0.2225	0.6813	−0.105	−0.2928	0.8239	−0.0876	0.1624		0.3302	0.9598	0.3069	−1.1131	0.0243	−1.8146	17.5201	1.1926	9.796	0.6495	−0.0137	
15th Center	0.0168	−1.7726	−0.8283	−0.6663	−0.1256	0.5306	0.3111		−0.1854	−0.2705	1.0725	0.0501	−3.7835	−0.0634	−0.6935	1.0361	−0.7715	0.8299	0.2224	
16th Center	−0.0554	−0.4717	−0.0756	0.032	0.4118	−0.0085	0.118		−0.757	−0.0234	0.9623	0.1031	−4.1496	1.526	−2.5147	0.7071	−7.7674	−0.1238	−6.9991	
17th Center	0.2334	0.1427	−0.7832	0.623	3.4933	−0.1823	−0.1855		−0.4816	−0.3675	−0.1266	−0.0592	0.449	−0.8812	4.688	0.3433	−1.9164	−0.9352	6.3565	
18th Center	−0.037	0.0676	0.0052	0.1383	0.3226	−0.1298	−0.3925		0.0384	0.0695	0.3908	−0.021	1.4033	−0.507	−1.1789	−0.5744	4.0255	0.1921	−0.6567	
19th Center	1.0126	−0.6111	−1.4189	0.8838	4.7513	−0.8125	0.9698		0.5533	−0.8134	−0.8803	0.706	−0.6277	−4.7769	0.4758	−1.4918	0.4319	−19.1898	0.5933	

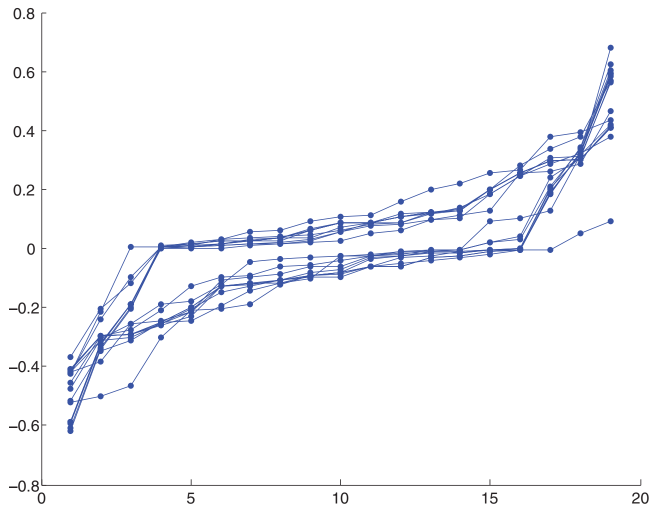

Fig. 14 Second eigen vector of the Laplacian associated with the dynamics of NAS. Eigen vectors for all 20 cases are shown.

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